Quality-spaces: problematic aspects

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this car is red at time t

- $\mathsf{RED}_t(\texttt{c\#1}) \leftrightarrow \mathsf{RED}(\texttt{c\#1}, t)$
- $\texttt{c\#1} ::_t \texttt{red}$

the color of this car is red at time t

- $\mathbf{color}(\texttt{c\#1}, t) = \texttt{red}$
- $\mathbf{color}(\texttt{c\#1}) ::_t \texttt{red}$

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this car is 800kg heavy at time t

- $\mathsf{800KG}_t(\texttt{c\#1})$
- $\texttt{c\#1} ::_t \texttt{800kg}$

the width of this car is 2m at time t

- $\mathbf{weight}(\texttt{c\#1}, t) = \texttt{800kg}$
- $\mathbf{weight}(\texttt{c\#1}) ::_t \texttt{800kg}$

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DOLCE adopts a reified version of

- $\textbf{color}(\texttt{c\#1}) ::_t \texttt{red} \wedge \mathbb{COLOR}(\texttt{red})$

(finite) set of functions defined between objects and *individual qualities* QT such that

- they are partial but cover the whole QT
- they are bijective: $\textbf{color}(x) = \textbf{color}(y) \leftrightarrow x = y$
- different functions are disjoint: $\textbf{color}(x) \neq \textbf{weight}(x)$

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note 1

- qualities QT are partitioned into *quality-kinds* $\mathbb{Q}_i$
- $q \curvearrowright a \wedge q \curvearrowright b \rightarrow a = b$ ($\curvearrowright$ is called *inherence*)
- $\mathbb{Q}_i q \rightarrow \exists x (q \curvearrowright x)$
- $\mathbb{Q}_i q \wedge \mathbb{Q}_i \bar{q} \wedge q \curvearrowright x \wedge \bar{q} \curvearrowright x \rightarrow q = \bar{q}$

(•) $q = \mathbf{q}_i(x) \leftrightarrow \mathbb{Q}_i(q) \wedge q \curvearrowright x$

(•) 1-1 correspondence between $\mathbf{q}_i$ and $\mathbb{Q}_i$

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note 2

- quality kinds/functions individuate classes of objects
- $\mathsf{Q}_i(x) \triangleq \exists q(q = \mathbf{q}_i(x)) \leftrightarrow \exists q(\mathbb{Q}_i \wedge q \curvearrowright x)$
- e.g., $\mathsf{COLORED}(x) \triangleq \exists q(q = \mathbf{color}(x))$

(•) Q_i do not partitionate objects
(•) metrology: a Q_i is called a *general property* (*quantity*)

note 3

(?) $\mathbf{color}(x) ::_t \texttt{scarlet} \wedge \mathbf{color}(x) ::_t \texttt{red}$

(•) we assume here that spaces are *complete* and properties are *fully-determined* and *disjoint*: at every t, each quality is an instance of an unique property in a **given** space

(•) given a space, $::_t$ is then functional (as 'ql_t' and 'loc_t')

(•) metrology: a fully-determined property, i.e., a property that can be predicated of an object, is called *individual property*

(•) here the structure of spaces is not considered

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given two different quality-functions $\mathbf{q}_i$ and $\mathbf{q}_k$

(?) $\mathbf{q}_i(x, t) = \mathbf{q}_k(y, t')$

(?) $\mathbf{q}_i(x) ::_t p \wedge \mathbf{q}_k(y) ::_{t'} p$

(•) are properties/attributes *local* or *non-local* to quality kinds?

DOLCE

- properties PT are partitioned in spaces $\mathbb{S}_i$
- $\mathbb{Q}_i(q) \wedge q ::_t p \rightarrow \mathbb{S}_j(p)$ $\quad$ $\mathbf{q}_i(x) ::_t p \rightarrow \mathbb{S}_j(p)$
- $\mathbb{S}_j(p) \wedge q ::_t p \rightarrow \mathbb{Q}_i(q)$ $\quad$ $\mathbb{S}_j(p) \wedge q ::_t p \rightarrow \exists x(q = \mathbf{q}_i(x))$

(•) 1–1 correspondence between *quality kinds* and *spaces*

(•) *unique* and *local* spaces

unicity

- $\mathbf{width}(x) ::_t \texttt{2mw} \rightarrow \neg\exists p(\mathbf{width}(x) ::_t p \land p \neq \texttt{2mw})$

locality

- $\mathrm{2MW}_t(x) \triangleq \mathbf{width}(x) ::_t \texttt{2mw}$
- $\mathrm{2MH}_t(x) \triangleq \mathbf{height}(x) ::_t \texttt{2mh}$

DOLCE-CORE

- properties PT are partitioned in spaces $\mathbb{S}_i$
- $\mathbb{Q}_i(q) \wedge q ::_t p \rightarrow \mathbb{S}_j(p)$
- $\mathbb{S}_j(p) \wedge q ::_t p \rightarrow \mathbb{Q}_i(q)$

(•) 1–*n* correspondence btw *quality kinds* and *spaces*

(•) *multiple* and local spaces

multiplicity

- $\mathbf{color}(x) ::_t \langle\texttt{r0.5g0.5b0.5}\rangle \wedge \mathbf{color}(x) ::_t \langle\texttt{h1s0.5b0.7}\rangle \wedge$
- $\langle\texttt{r0.5g0.5b0.5}\rangle \neq \langle\texttt{h1s0.5b0.7}\rangle$

locality

- $\mathsf{GREY}_t^{\mathsf{rgb}}(x) \triangleq \mathbf{color}(x) ::_t \langle\texttt{r0.5g0.5b0.5}\rangle$
- $\mathsf{GREY}_t^{\mathsf{hsb}}(x) \triangleq \mathbf{color}(x) ::_t \langle\texttt{h1s0.5b0.7}\rangle$

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locality

- there is a relation btw `2mw` and `2mh` ?
- there is a relation btw ⟨`r0.5g0.5b0.5`⟩ and ⟨`h1s0.5b0.7`⟩ ?

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- properties PT are partitioned in spaces $\mathbb{S}_i$
- $\mathbb{Q}_i(q) \land q ::_t p \rightarrow \mathbb{S}_j(p)$
- $\mathbb{S}_j(p) \land q ::_t p \rightarrow \mathbb{Q}_i(q)$

(•) *n*–1 correspondence between *quality kinds* and *spaces*

(•) unique *but non-local* spaces

unicity

(•) $\textbf{width}(x) ::_t \texttt{2m} \rightarrow \neg\exists p(\textbf{width}(x) ::_t p \wedge p \neq \texttt{2m})$

non-locality

- $\mathsf{2MW}_t(x) \triangleq \textbf{width}(x) ::_t \texttt{2m}$
- $\mathsf{2MH}_t(x) \triangleq \textbf{height}(x) ::_t \texttt{2m}$

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- is ‘being 2m’ a property ?

option 1

- ‘being 2m’ is a property
- it is an abstraction from, a disjunction of, ‘being 2m wide’, ‘being 2m high’, ‘being 2m long’, . . .
- (note that width-, height-, length-, etc. spaces are isomorphic)

- does this require spaces composed by (isomorphic) spaces, or more generally, a complex abstraction mechanism among spaces?
- are disjunctions of properties still (fully-specified) properties?

option 2

- ‘being 2m long’ (`2ml`) instead of ‘being 2m’ (`2m`)
- qualities can have qualities
- $\mathsf{2ML}_t(q) \triangleq \mathbf{length}(q) ::_t \texttt{2ml}$
- $\mathsf{2MW}_t(x) \triangleq \mathbf{length}(\mathbf{width}'(x)) ::_t \texttt{2ml}$
- $\mathsf{2MH}_t(x) \triangleq \mathbf{length}(\mathbf{height}'(x)) ::_t \texttt{2ml}$
- $\mathbf{width}(x) \triangleq \mathbf{length}(\mathbf{width}'(x))$, $\mathbf{height}(x) \triangleq \mathbf{length}(\mathbf{height}'(x))$

option 3 (intuition)

- the meter can be used to measure both the *height* and the *width* of an object
- one just uses it in different ways
- the measurement procedure is relevant here

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- no time from here: equivalence classes are a problem measurement in the *wide sense* (also called *evaluation*)

■ “assignment of symbols to attributes of objects (...) in a such way to represent them, or to describe them.” [Finkelstein 2003]

(•) *nominal* evaluation is included: symbols are not necessarily structured *evaluation* along a quality type/attribute $\mathbb{Q}_i$

- $\mathbf{q}_{\mathbf{E}_i} : \mathcal{Q}_i \rightarrow \mathrm{SY}$ $\mathcal{Q}_i(x) \triangleq \exists q (q = \mathbf{q}(x) \wedge \mathbb{Q}_i(q))$

classification along a quality type/attribute $\mathbb{Q}_i$

- $\mathbf{q}_{\mathbf{C}_i} : \mathcal{Q}_i \rightarrow \mathrm{PR}$ $\mathbf{q}_{\mathbf{C}_i}(x) = p \textit{ iff } \mathbf{q}_i(x) :: p$

symbolisation of properties

- $\mathbf{q}_{\mathbf{S}_i} : \mathrm{PR} \rightarrow \mathrm{SY}$
- $\mathbf{q}_{\mathbf{E}_i}(x) = \mathbf{q}_{\mathbf{S}_i}(\mathbf{q}_{\mathbf{C}_i}(x))$ / with qualities $\mathbf{q}_{\mathbf{E}_i}(x) = \mathbf{q}_{\mathbf{S}_i}(\mathbf{q}_{\mathbf{C}_i}(\mathbf{q}(x)))$

- symbols (linguitic nature) SY are partitioned in $\mathbb{V}_i$

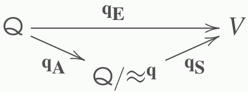
(?) $\mathbb{S}_i(p) \wedge p \triangleright v \rightarrow \mathbb{V}_j(v)$

(?) $\mathbb{V}_j(v) \wedge p \triangleright v \rightarrow \mathbb{S}_i(p)$

(•) $\mathbf{q}_{\mathbf{C}i}$ is defined only for unique space

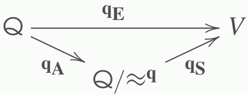
(•) $\mathbf{q}_{\mathbf{E}i}$ is defined only for unique space + unique symbol

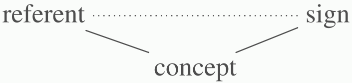
(•) this means that in the general case $\mathbb{V}_i$ is not enough to individuate the quality-kind along with the object is evaluated in the particular case of unique space and unique symbol



- $x \in \|r\|_{\approx^{\mathbf{q}}}$ *iff* $\mathbf{q}(x) :: \mathtt{p} \wedge \mathbf{q}(r) :: \mathtt{p}$
- (•) 1-1 relation btw eq. classes and *non-empty* properties
- (•) intensional properties vs. extensional classes [res. nomin.]
- (•) different general properties can share eq. classes
- $\mathbf{q_A}$ is surjective, $\mathbf{q_S}$ is injective

in the particular case of unique space and unique symbol





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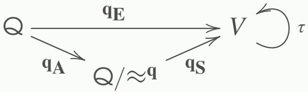
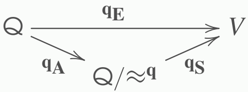
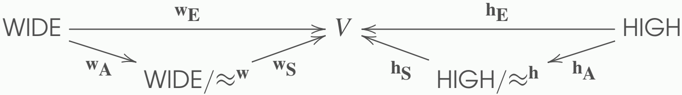


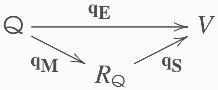
- are *values* symbols or equivalent classes of symbols ?



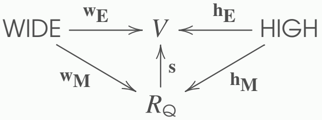
(•) if $\mathbf{q_S}$ is bijective then $\mathbf{q}_{\mathbf{C}_i}$ and $\mathbf{q}_{\mathbf{E}_i}$ are equi-informative

- how to justify the interesting case (assuming $\mathbb{S}_{\mathsf{W}} \neq \mathbb{S}_{\mathsf{H}}$) ?





- $R_{\mathsf{Q}} \subseteq \mathsf{Q}$: set of *reference* objects in 1-1 correspondence with the eq. classes in $\mathsf{Q}/\!\approx^{\mathbf{q}}$ and (non-empty) properties in $\mathbb{S}$
- $\mathbf{q_M}$ represents a comparison procedure related to Q
- $x \approx^{\mathbf{q}} y$ *iff* $\mathbf{q_M}(x) = \mathbf{q_M}(y) = r$
- if $r \in R_{\mathsf{Q}}$ then $\mathbf{q_M}(r) = r$ and $r \approx^{\mathbf{q}} r$



- if $r \in R_{\mathsf{Q}}$ then $\mathbf{w_M}(r) = r$ and $\mathbf{h_M}(r) = r$
  - the procedures $\mathbf{w_M}$ and $\mathbf{h_M}$ are different
- in the case of unique space for both height and width:
  - $\mathbf{w_M}(r) = r$ *iff* $\mathbf{w}(r) :: p_r$ and $\mathbf{h_M}(r) = r$ *iff* $\mathbf{h}(r) :: p_r$
  - **our reference meter is both 1m hight and 1m width**

- the measurement procedure is not really represented
- maybe, assuming only local spaces, values can be modeled by [reifications of] classes of properties linked by an additional primitive, a sort of *calibration*